Comments on "Polygonal Coaxial Line with Round Center Conductor"

MOHAMED D. ABOUZAHRA, MEMBER, IEEE

In the above paper, [1] the author presented an expression for the attenuation constant α of a polygonal coaxial line with round center conductor. Apart from a typographical error in (20b), other more serious errors are believed to exist. First, the f_N expression given in (20b) should be corrected to become

$$f_N = 1.55694 + 0.024683 N \int_0^{\pi/N} (\sec\theta)^{2N} d\theta - \frac{N}{\pi} \int_0^{\pi/N} \log(\sec\theta)^{2N} d\theta.$$

In addition, it is believed that the values of f_N reported in the above paper for the special cases $N = 3$, 4, 5, and 6 are also in error. Indeed, the correct results should read as follows:

$$f_N = 0.9211884 \text{ for } N = 3$$
$$f_N = 0.9475403 \text{ for } N = 4$$
$$f_N = 1.0619316 \text{ for } N = 5$$
$$f_N = 1.1498117 \text{ for } N = 6.$$

By examining these numbers it becomes very clear that the corrected expression of f_N is itself in error. This is because one would expect f_N to increase, and not exceed 1, as N increases. Indeed, f_N should reach 1 only as $N \to \infty$, which corresponds to the regular coaxial line case as Mr. Lin indicated in the closing lines of Section V of his paper. However, the expression of f_N, as it stands now, is found to become equal to 1.63448, rather than 1, as $N \to \infty$. The derivation of f_N, as reported in the above paper, has been checked and no error is detected. This, in turn, suggests that the expression of $|E|^2$ may be in error.

Finally, it is worthwhile to point out that both integrals which appear in f_N are able to be done in closed form. The first integral is elementary and may be written as follows:

$$\int_0^{\pi/N} (\sec\theta)^{2N} d\theta = \sum_{r=0}^{N-1} \frac{(N-1)! \tan^{2r+1}\left(\frac{\pi}{N}\right)}{(N-r-1)! r! (2r+1)}.$$

In addition, the second integral is related to Clausen's integral and may be written as [1]

$$\int_0^{\pi/N} \log(\sec\theta)^{2N} d\theta = 2\pi \log 2 - N \mathrm{Cl}_2\left(\frac{N-2}{N}\pi\right)$$

where Clausen's integral is defined as

$$\mathrm{Cl}_2(x) = -\int_0^x \log\left(2\sin\frac{\theta}{2}\right) d\theta = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}.$$

This integral is tabulated and one might find many of its useful properties in Lewin's book on Polylogarithms.

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[1] W. Lin, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 545–550, June 1985

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Reply[2] *by W. Lin*[3]

Thanks to M. D. Abouzahra for pointing out the typographical error and the erroneous numerical values of f_N, $N = 3$, 4, 5, 6, in my paper.[1] The corrected results are given as follows: the first term of f_N should read 1.55694 and f_N for various N are listed:

$N = 3$	$f_N = 0.92217565$
$N = 4$	$f_N = 0.94757545$
$N = 5$	$f_N = 1.06190240$
$N = 6$	$f_N = 1.14990723$
$N = 13$	$f_N = 1.40516210$
$N = 14$	$f_N = 1.42127669$
$N = 15$	$f_N = 1.43527651$
$N = 16$	$f_N = 1.44755161$
$N = 23$	$f_N = 1.50388527$
$N = 24$	$f_N = 1.50927532$
$N = 25$	$f_N = 1.51423812$
$N = 26$	$f_N = 1.51882243$
$N = 33$	$f_N = 1.54318583$
$N = 34$	$f_N = 1.54585266.$

As $N \to \infty$, f_N does approach a constant, but not unity, as we have expected due to the approximate nature of our solution to the problem of the polygonal coaxial line.

Indeed, both integrals in f_N in our paper are elementary [2]

$$I_{2N} = \int \frac{dx}{\cos^{2N} x} = \frac{\sin x}{2N-1} \left\{ \sec^{2N-1} x + \sum_{k=1}^{N-1} \frac{2^k (N-1)(N-2)\cdots(N-k)}{(2N-3)(2N-5)\cdots(2N-2k-1)} \sec^{2N-2k-1} x \right\}$$

$$I_2 = \tan x \quad I_4 = \frac{1}{3}\tan^3 x + \tan x$$

$$I_6 = \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x + \tan x$$

$$I_8 = \frac{1}{7}\tan^7 x + \frac{3}{5}\tan^5 x + \tan x + \tan x, \ldots$$

$$\int_0^u \ln\sec x\, dx = L(u)$$

$$L(x) = x\ln 2 - \frac{1}{2}\sum_{k=1} (-1)^{k-1} \frac{\sin 2kx}{k^2}.$$

[2] Manuscript received August 12, 1985

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Those given by Abouzahra are correct except that the last summation should start from $n=1$, not from $n=0$, which will lead to divergence.

References

[1] L. Lewin, *Polylogarithms and Associated Functions.* New York: Elsevier, 1981, ch. 4.

[2] I. S. Gradshteyn and I. M Ryzhik, *Table of Integrals, Series, and Products* New York: Academic Press, 1980, p. 20, eqs. 2.519 and 4 224, p 4, eq. 8.261.

Correction to "Formulation of the Singular Integral Equation Technique for Planar Transmission Lines"

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In the above paper,[1] it has been wrongly concluded that (17) and (18), which represent the energy coupling between a pair of complex modes, mean that each mode cannot exist alone.

Exciting only one mode of a pair of complex modes is, in principle, possible[2] because a mode of propagation is a possible solution of Maxwell's equations inside the guiding structure which satisfies the boundary and/or the radiation conditions. The difference between such a mode and other usual modes is that it carries, by itself, neither active nor reactive power (see (17)).

Propagating modes carry active power, whereas evanescent modes carry reactive power. Only *modes at cutoff* carry neither active nor reactive power. We can then conclude that one mode of a pair of complex modes behaves in this aspect like a mode at cutoff, provided that the other mode of the same pair is not excited. When both modes are excited, the situation is different. Now they interact together (see (18)) so that both carry reactive power.

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[1]A. S. Omar and K. Schünemann, *IEEE Trans. Microwave Theory Tech*, vol. MTT-33, pp. 1313–1322, Dec. 1985

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[2]As has been pointed out by Prof. Piefke of the Technische Universität Darmstadt.